2024-2025
Fall Semester

Course of Power System Analysis
Initial Transient of the Short-Circuit Current

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Outline

Initial transient of the short-circuit current

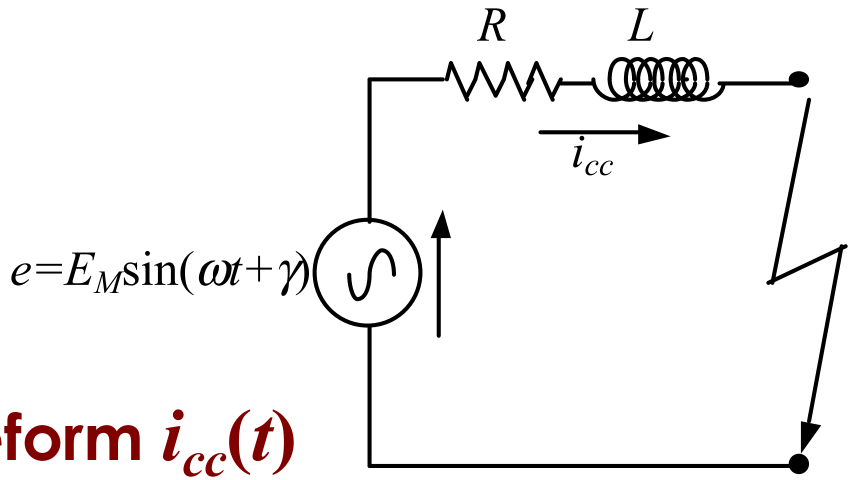
Switching and protection devices for electrical networks

Initial transient of the short-circuit current

Hypothesis: we assume the **generators to be distant from the short circuit location and without any influence associated to their time-varying behaviour after a short circuit**→ the Thévenin equivalent electrical parameters are **constant**. The equivalent circuit can be modelled as shown below.

The differential equation driving this circuit is:

$$E_M \sin(\omega t + \gamma) = Ri_{cc}(t) + L\frac{di_{cc}(t)}{dt}$$



Objective: determine the waveform $i_{cc}(t)$

Initial transient of the short-circuit current

The **homogeneous equation** associated with the previous equation is:

$$Ri_{cc}(t)+L\frac{di_{cc}(t)}{dt}=0$$

The solution is:

$$i_0(t)=K_1e^{-\frac{R}{L}t}=K_1i_1(t)$$

(where K_1 is a constant)

A **particular integral** of the differential equation driven by the DC transient, in the case of **sinusoidal excitation**, is given by:

$$i_p(t)=K(t)e^{-\frac{R}{L}t}=K(t)i_1(t)$$

Initial transient of the short-circuit current

The value of $K(t)$, capable of providing a solution to the original differential equation, can be used to **determine a particular integral**

$$E_M \sin(\omega t + \gamma) = RK(t) i_1(t) + L \frac{dK(t)}{dt} i_1(t) + LK(t) \frac{di_1(t)}{dt}$$

$$E_M \sin(\omega t + \gamma) = K(t) \left(R i_1(t) + L \frac{di_1(t)}{dt} \right) + L \frac{dK(t)}{dt} i_1(t)$$

as long as $\boldsymbol{i_1}$ **is a solution of the associated homogeneous equation**, the first term of the second expression is equal to zero, therefore:

$$E_M \sin(\omega t + \gamma) = L \frac{dK(t)}{dt} i_1(t)$$

$$K(t) = \frac{E_M}{L} \int e^{\frac{R}{L}t} \sin(\omega t + \gamma)\, dt$$

Initial transient of the short-circuit current

If we integrate the previous equation by parts, we obtain

$$K(t) = \frac{E_M}{\omega L} e^{-\frac{R}{\omega L}\gamma} \frac{R\omega L}{R^2 + \omega^2 L^2} e^{\frac{R}{\omega L}(\omega t + \gamma)} \left[\sin(\omega t + \gamma) - \frac{\omega L}{R} \cos(\omega t + \gamma) \right] =$$

$$= \frac{E_M R}{R^2 + \omega^2 L^2} e^{\frac{R}{L}t} \left[\sin(\omega t + \gamma) - \frac{\omega L}{R} \cos(\omega t + \gamma) \right]$$

The following parameters are defined:

$$Z_{cc} = \sqrt{R^2 + \omega^2 L^2}, \mathrm{tg}\,\varphi_{cc} = \frac{\omega L}{R}, \cos\varphi_{cc} = \frac{R}{Z_{cc}}$$

Initial transient of the short-circuit current

We obtain:

$$K(t) = \frac{E_M}{Z_{cc}} \cos\varphi_{cc} e^{\frac{R}{L}t} \left[\sin(\omega t + \gamma) - \frac{\sin\varphi_{cc}}{\cos\varphi_{cc}} \cos(\omega t + \gamma) \right] =$$

$$= \frac{E_M}{Z_{cc}} e^{\frac{R}{L}t} \left[\sin(\omega t + \gamma)\cos\varphi_{cc} - \cos(\omega t + \gamma)\sin\varphi_{cc} \right] =$$

$$= \frac{E_M}{Z_{cc}} e^{\frac{R}{L}t} \sin(\omega t + \gamma - \varphi_{cc})$$

Therefore, the **particular integral** is:

$$i_p(t) = K(t) i_1(t) = \frac{E_M}{Z_{cc}} e^{\frac{R}{L}t} e^{-\frac{R}{L}t} \sin(\omega t + \gamma - \phi_{cc})$$

Initial transient of the short-circuit current

The solution becomes:

$$i_{cc}(t) = i_0(t) + i_p(t) = K_1 e^{-\frac{R}{L}t} + \frac{E_M}{Z_{cc}} \sin(\omega t + \gamma - \varphi_{cc})$$

If we assume that $t=0$ → $i_{cc}(0)=0$ we can determine the value of the constant K_1

$$0 = K_1 + \frac{E_M}{Z_{cc}} \sin(\gamma - \varphi_{cc})$$

Therefore, by substitution:

$$i_{cc}(t) = -\frac{E_M}{Z_{cc}} e^{-\frac{R}{L}t} \sin(\gamma - \varphi_{cc}) + \frac{E_M}{Z_{cc}} \sin(\omega t + \gamma - \varphi_{cc})$$

Initial transient of the short-circuit current

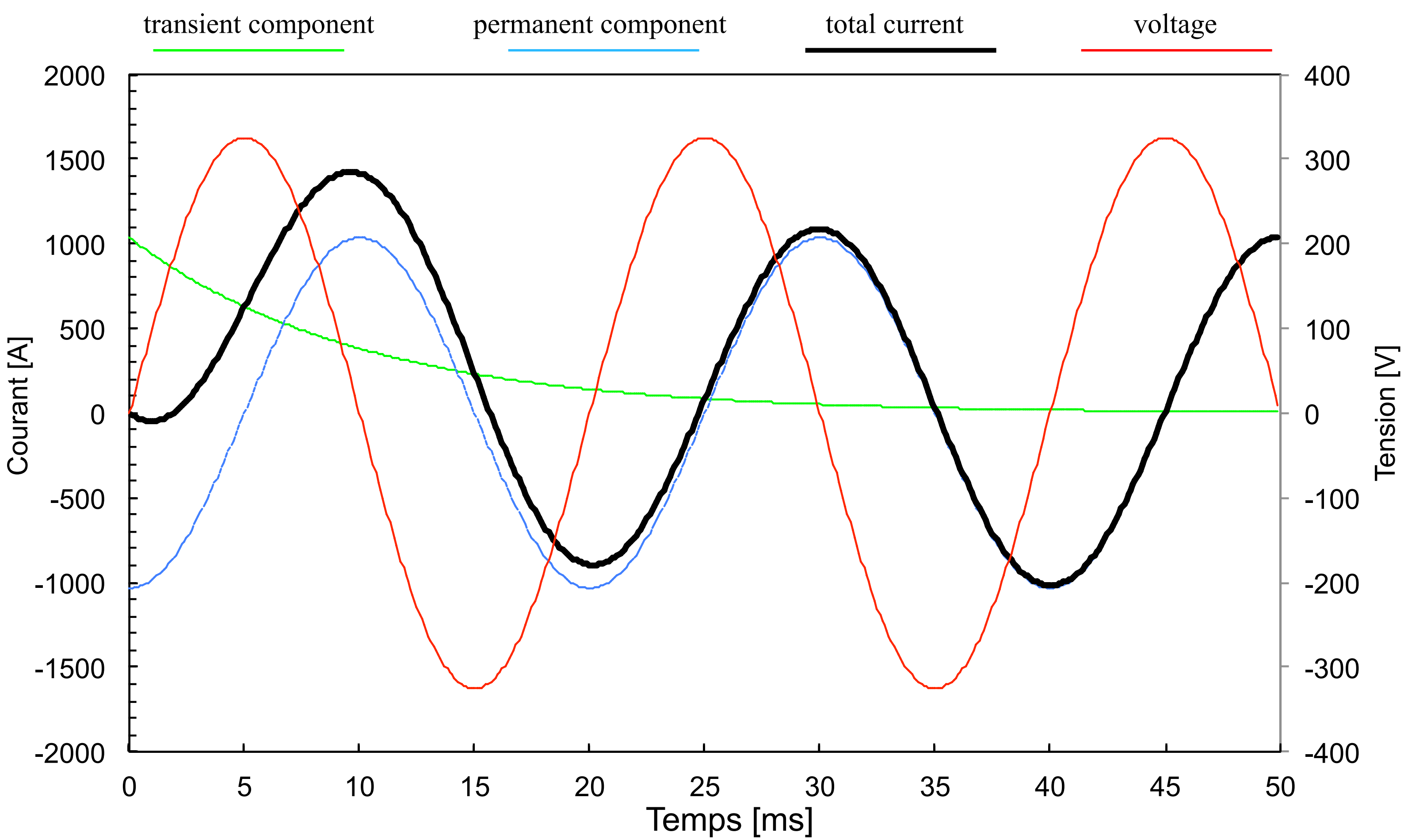
$$i_{cc}(t) = -\frac{E_M}{Z_{cc}} e^{-\frac{R}{L}t} \sin(\gamma - \varphi_{cc}) + \frac{E_M}{Z_{cc}} \sin(\omega t + \gamma - \varphi_{cc})$$

<u>Observation</u>: the short-circuit current results from **superposition** of a **sinusoidal (permanent) component** and an **aperiodic (transient) component.**

The term E_M / Z_{cc} is the **maximum value (amplitude)** of the **permanent component** of the short-circuit current, indicated by I_M.

Initial transient of the short-circuit current

$$i_{cc}(t) = -\frac{E_M}{Z_{cc}} e^{-\frac{R}{L}t} \sin(\gamma - \varphi_{cc}) + \frac{E_M}{Z_{cc}} \sin(\omega t + \gamma - \varphi_{cc})$$



Initial transient of the short-circuit current

Observations:

1. The maximum value of the short-circuit current (peak I_P) **depends on the time at which the short circuit is established**. If $\gamma = \varphi_{cc}$, the transient component is zero, if $\gamma < \varphi_{cc}$ the transient component is **positive**, if $\gamma > \varphi_{cc}$ the transient component is **negative**.
2. The duration of the transient and peak current I_P depend on the R and L parameters of the circuit, i.e., $\cos\varphi_{cc}$. The exponential term has a time constant $T_0 = L / R = \tan\varphi_{cc} / \omega$. Therefore, if $R=0$ ($\cos\varphi_{cc}=0$) $T_0 \rightarrow \infty$ and $I_P=2I_M$. **Thus, the more inductive the circuit, the greater the value of the aperiodic component and the greater the peak short-circuit current.**

Initial transient of the short-circuit current

To consider the aperiodic component of the short-circuit current, we introduce the factor χ which gives the **ratio between the peak value and the maximum value** of the symmetrical component of the short-circuit current**:**

$$I_P = \sqrt{2}\chi I_{cc} = \chi I_M$$

where I_{cc} is the **RMS value of the symmetrical component** of the short-circuit current.

Problem: compute the parameter χ

Initial transient of the short-circuit current

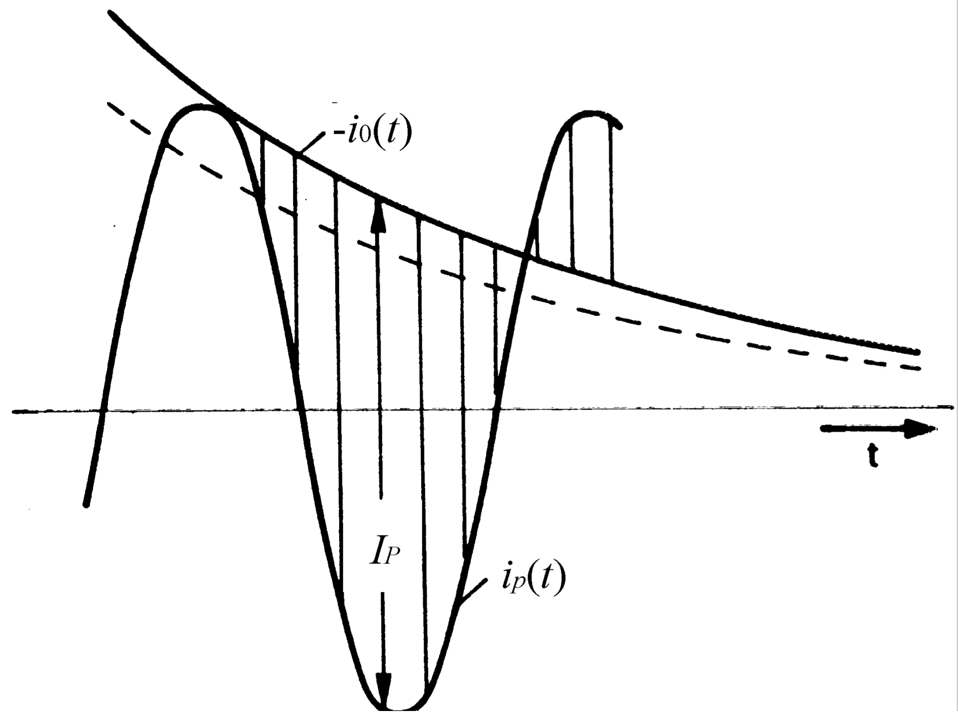
To determine the value of the parameter χ we should **maximize the value of the current** $i_{cc}(t)$ as a function of the parameter γ (i.e., the instant at which the short-circuit begins). **For each value of the phase φ_{cc} there is only one value γ for which the peak value is reached.**

Initial transient of the short-circuit current

Considerations:

- for $t = 0$, in order to meet the condition $i(0)=0$, it must be true that $i_0(0)+i_P(0)=0$ (initial values of the aperiodic and permanent components equal and opposite);

- **the maximum amplitude of $i(t)$, equal to the sum of $i_P(t)$ and $-i_0(t)$, is attained if the waveform $-i_0(t)$ is tangent to $i_P(t)$ for $t=0$.**



Initial transient of the short-circuit current

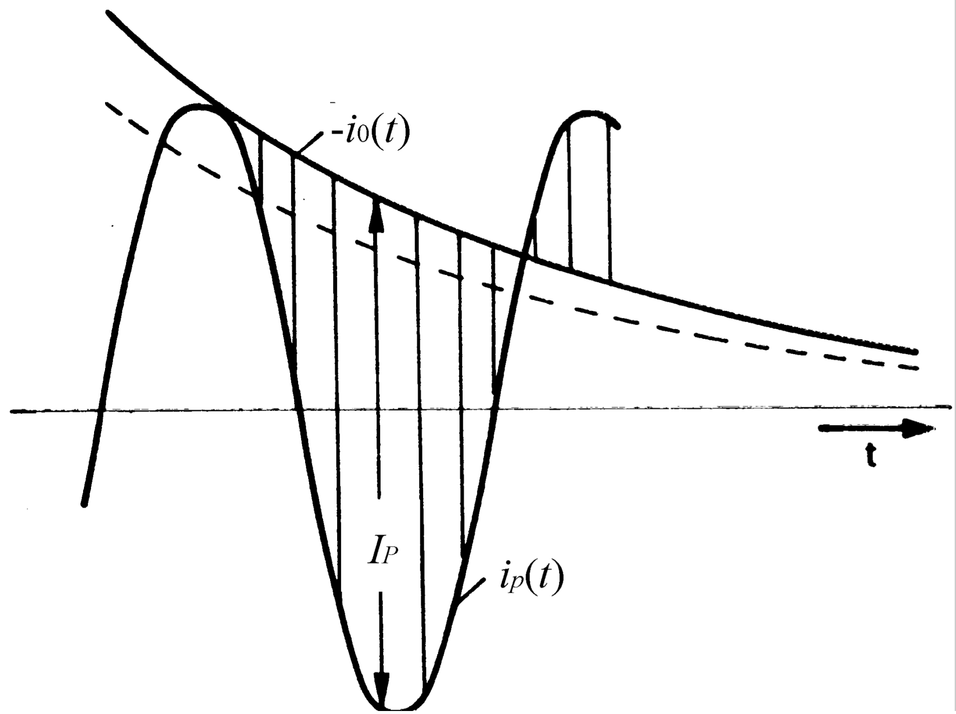
Therefore:

$$-\left[\frac{di_0(t)}{dt}\right]_{t=0} = \left[\frac{di_p(t)}{dt}\right]_{t=0} \Rightarrow$$

$$-\frac{1}{T_0}\sin(\gamma-\varphi_{cc}) = \omega\cos(\gamma-\varphi_{cc})$$

If the relation $\omega T_0 = \tan\varphi_{cc}$ is satisfied:

$$\tan(\gamma-\varphi_{cc}) = -\tan\varphi_{cc}$$



This equation is verified for $\gamma=0$ and also $\gamma=\pi$ → **the largest value of I_P** is reached if the short-circuit occurs **when the voltage crosses zero, for any value of φ_{cc}.**

Initial transient of the short-circuit current

The time $\hat{t}$ at which the value I_P is reached occurs if $\frac{di_{cc}(t)}{dt}=0$ i.e., for $\gamma=0$ or for $\gamma=\pi$ in the derivation of the equation:

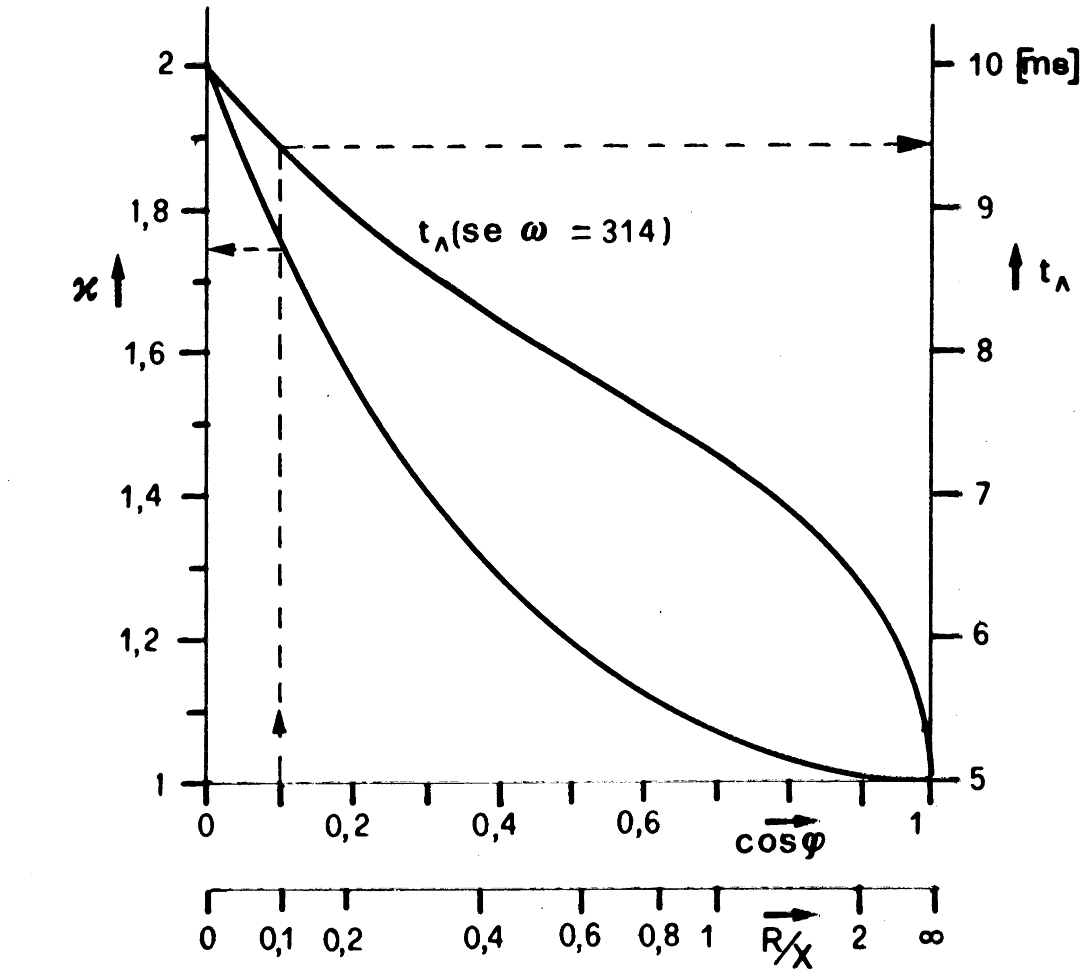
$$\frac{di_{cc}(t)}{dt}=\frac{d}{dt}\left[-\frac{E_M}{Z_{cc}}e^{-\frac{R}{L}t}\sin(\gamma-\phi_{cc})+\frac{E_M}{Z_{cc}}\sin(\omega t+\gamma-\phi_{cc})\right]=0$$

$$\cos\left(\omega\hat{t}-\varphi_{cc}\right)-e^{-\frac{R}{L}\hat{t}}\cos\varphi_{cc}=0$$

The value $\hat{t}$ enables calculation of the values I_P and χ, if subsituted in the equation for $i_{cc}(t)$.

Initial transient of the short-circuit current

The figure shows the relation between $\hat{t}$ and χ as a **function of the ratio R/X or $\cos\varphi_{cc}$**



Initial transient of the short-circuit current

Summary of the descriptive parameters of the initial transient of the short-circuit current:

- $I_M = E_M / Z_{cc}$ maximum value of the permanent component of the short-circuit current;
- $T_0 = L / R$ time constant of the transient component of the short-circuit current;
- I_{cc} RMS value of the permanent component of the short-circuit current;
- I_P peak value of the short-circuit current;
- $\cos\varphi_{cc}$ power factor in short-circuit conditions
- χ relation between I_P and $\sqrt{2}I_{cc}$

Initial transient of the short-circuit current

Observation: knowing the value of the parameter χ (and therefore the value I_P) enables evaluation of **thermal stresses directly after the start of the short-circuit**.

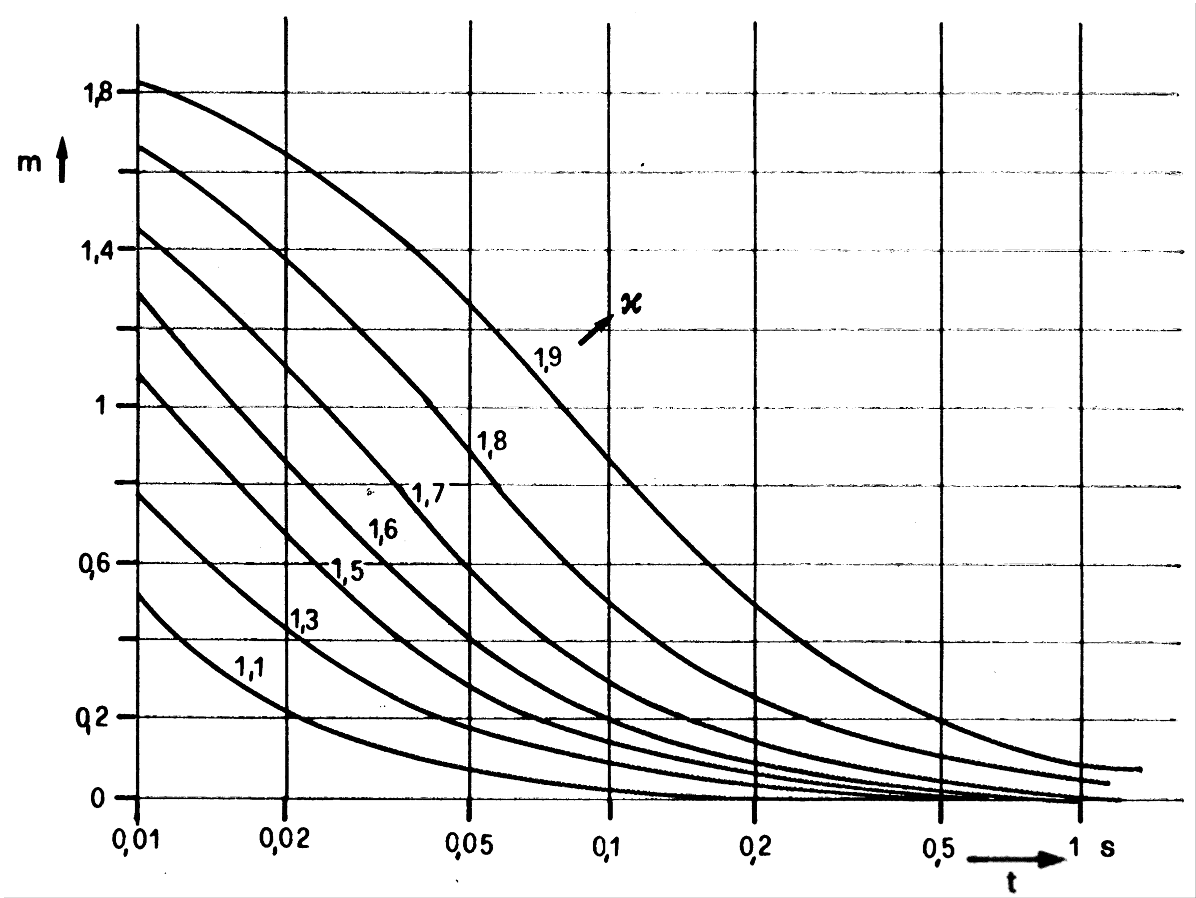
During this period, the aperiodic component $i_0(t)$ is not damped and therefore its contribution to Joule's specific energy is considerable, in addition to the contribution of the periodic component. Network heating upstream of the short circuit is highly dependent on the specific energy. **It is possible to determine an RMS current value equivalent in terms of Joule integral to $i_{cc}(t)$.**

Initial transient of the short-circuit current

This value is given by:

$$I_{eff} = \frac{I_M}{\sqrt{2}}\sqrt{m+1}$$

the coefficient m depends on $i_0(t)$, and therefore on parameter χ and the duration of the short-circuit. The figure shows the value of m as a function of these two quantities.



Outline

Initial transient of the short-circuit current

Switching and protection devices for electrical networks

Switching and protection devices for electrical networks

The nominal characteristics of devices for operation and protection of electrical networks (i.e., circuit breakers) are:

- Rated voltage U_r
- Maximum voltage U_m
- Level of insulation (steady-state voltage U_d at $50\ \mathrm{Hz}$ and impulse voltage U_p)
- Nominal current I_r
- Nominal acceptable current for short duration I_k
- Nominal duration of short-circuit t_k

Switching and protection devices for electrical networks

The **nominal voltage** is the voltage assigned by the manufacturer, indicated by U_r in product standards and by U_n in network standards.
The **maximum voltage** is represented by U_m and represents the highest voltage value at any point on the network under normal operating conditions.
The maximum voltage is necessary to evaluate the **level of insulation of electrical devices.** The insulation level of an electrical device is linked to the rated voltage, and is assessed using **two reference values:**

- The **steady-state operating voltage** at 50 Hz U_d applied for 60 s;
- the **impulse voltage** U_p, for the waveform $1.2/50$ μs

Switching and protection devices for electrical networks

The **nominal current** for short duration I_k is the RMS value of the short-circuit current that can flow through the device during the time t_k.

Common values of I_k are 8, 10, 12.5, 16, 20, 25 kA.

The value t_k is commonly equal to **1 second**, but it notably must be **larger than the time necessary for the protection to interrupt the short-circuit.**

Switching and protection devices for electrical networks

The additional characteristics for **circuit breakers** in medium-voltage networks are as follows:

- **interruption time**: time needed for the circuit breaker to completely interrupt the short-circuit, given by the sum of the opening time (i.e., the time for the circuit breaker to open the contacts) and the arc extinction time.
- **rated breaking capacity**: the highest power value that the circuit breaker is capable of interrupting under nominal conditions.
- **restoring capacity**: the highest power value under which the circuit breaker is capable of closing when the circuit is re-established.